Reconstructing SUSY Theories at LHC and LC

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Overview

- Motivation
- Top-down analysis (LHC)
    - Top-down fit, hypothesis test
    - LHC observables, exp & th uncertainties
    - Discrimination of models
- Bottom-up analysis (LHC+LC)
    - Typical LC precisions
    - Necessary LHC input
    - Bottom-up reconstruction of high-scale parameters
- Conclusions

Motivation

- General SUSY phenomenology involves more than 100 new parameters

  → from the theoretical point of view: not satisfactory

  → from the practical point of view: too many possibilities

- Models of SUSY breaking provide relations between the SUSY breaking parameters at some high-energy scale, such as the GUT scale. Many schemes exist:

  SUGRA, AMSB, GMSB, gaugino mediation,
  string inspired models (dilaton/moduli) ......

- Different models feature different (though sometimes similar) characteristics of the sparticle spectrum

  → few parameters, predictive, good for benchmarking, etc.

Motivation

Once SUSY is discovered, we will want to relate our observations to the high-scale structure of the theory !

$\longrightarrow$ turn sparticle properties measured at the EW/TeV scale into SUSY-breaking parameters at the high-energy scale

- renormalization group evolution
- sophisticated computational tools
- good strategy, relevant measurements
- careful assessment of uncertainties

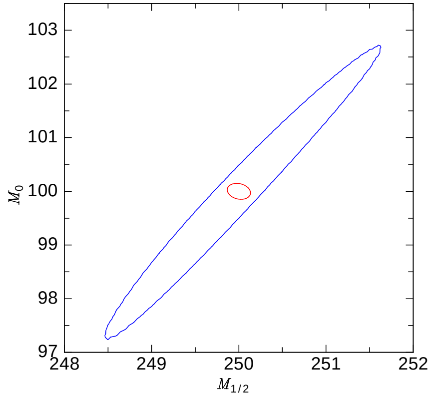
Part of this shall be addressed in this talk ....

LHC:
top-down analysis

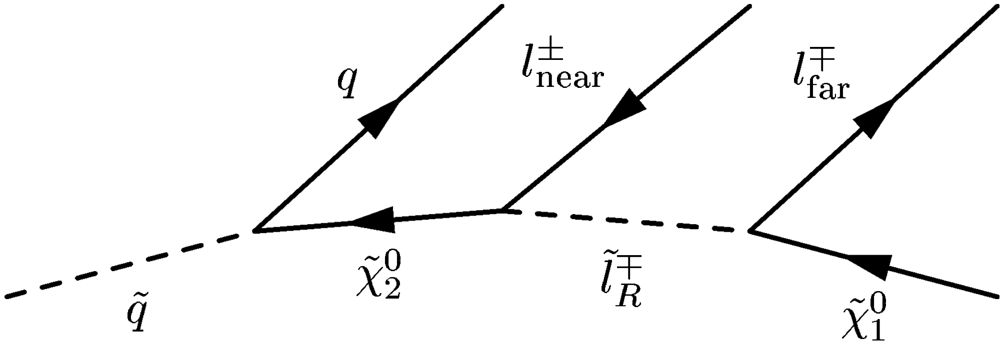
Top-down fit

Quite simple idea: take all available data and fit a particular model, e.g. mSUGRA, to them. [e.g. ATLAS Physics TDR]

- A χ^2 analysis gives the 1-, 2-, 3-σ errors on the input parameters.
- The $\chi^2_{min}/n.d.o.f.$ is a measure for the goodness of the fit (probabiliy that the model is wrong) $\rightarrow$ hypothesis test
- Advantage: works with any amount and precision of experimental data $\rightarrow$ convenient for analyses at LHC



LHC: cascade decays



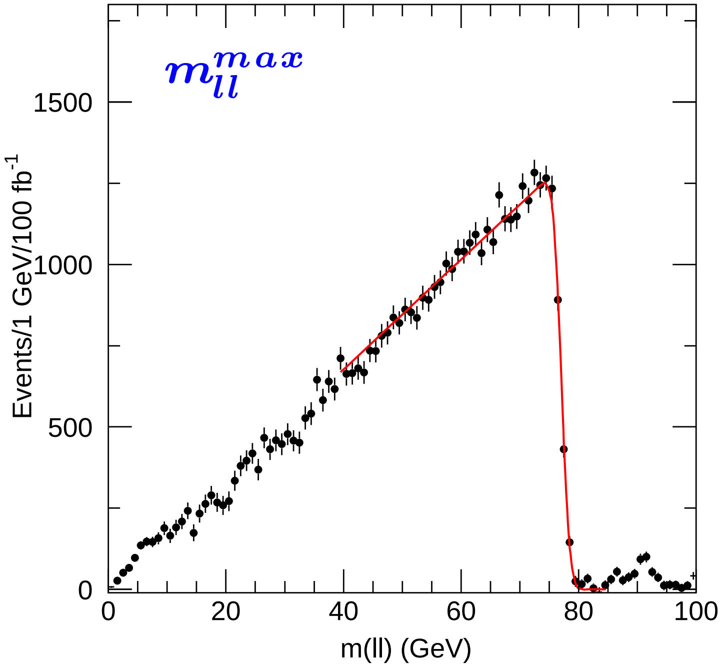
Can construct various kinematic endpoints:

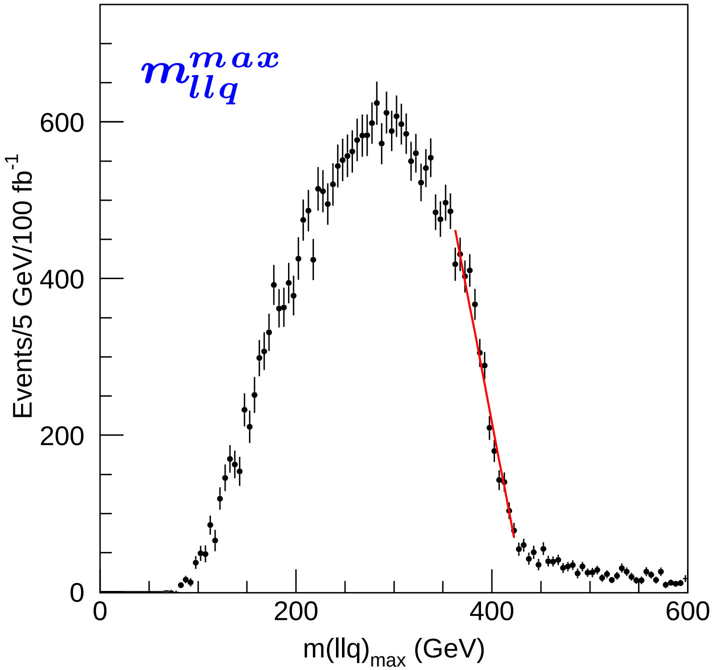
$l^+ l^-$	edge	m_{ll}^{max}
$l^+ l^- q$	edge	m_{llq}^{max}
$l^+ l^- q$	threshold	m_{llq}^{min}
$l^{\pm} q$	high-edge	m_{lq}^{high}
$l^{\pm} q$	low-edge	m_{lq}^{low}

[B.C. Allanach et al., hep-ph/0007009]

Case study of SPS1a

$$m_0 = 100, \quad \tan\beta = 10, \quad \mu > 0,$$
$$m_{1/2} = 250, \quad A_0 = -100, \quad m_t = 175$$





[G. Polesello]

LHC observables for SPS1a

	m_{ll}^{max}	m_{llq}^{max}	m_{llq}^{min}	m_{lq}^{high}	m_{lq}^{low}
ideal	80.64	454.0	216.8	397.2	325.6
Δ_{exp}	0.08	4.5	2.6	3.9	3.1

	$m_{\tau\tau}^{max}$	$m_{ll}^{max}(\tilde{\chi}_4^0)$	$m_{llb}^{min}(\tilde{b}_1)$
ideal	83.4	283.4	195.9
Δ_{exp}	5.1	2.3	4.1

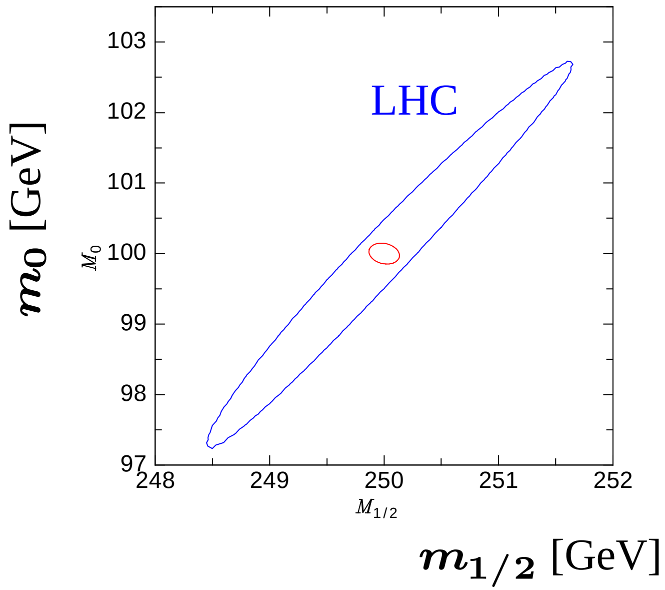
plus some mass differences, m_{h^0} ....

'ideal' values calculated with Spheno2.2.0

Δ_{exp} from ATLAS analysis, G. Polesello et al.

Fit of mSUGRA to SPS1a

Top-down fit of mSUGRA parameters to LHC data at SPS1a:



$$
\begin{aligned}
m_0 &= 100 \pm 2.8 \\
m_{1/2} &= 250 \pm 2.1 \\
A_0 &= -100 \pm 34 \\
\tan\beta &= 10.0 \pm 1.8
\end{aligned}
$$

$$\chi^2_{min}/14\, d.o.f. \sim 10^{-5}$$

Looks very promising, HOWEVER

- this cannot exclude other models, especially non-univ. scalar masses
- no theoretical uncertainties are taken into account

[Allanach et al., hep-ph/0403133]

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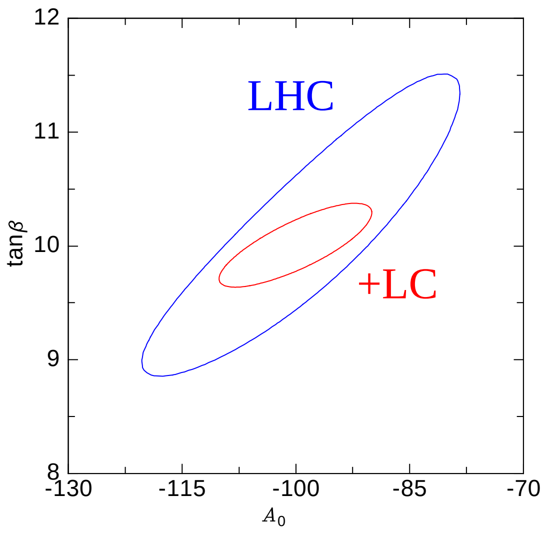

$\tan\beta$

A_0 [GeV]

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Theoretical uncertainties

.... arise from truncating the pertubation series of

- the RG evolution of the $\overline{\mathrm{DR}}$ SUSY-breaking parameters,
- the relation between $\overline{\mathrm{DR}}$ parameters and physical masses.

Moreover, uncertainties in the SM input parameters (α_s, m_t, $m_b, \ldots$) can have non-negligible effects on the SUSY spectrum.

State-of-the-art in SUSY spectrum computations:

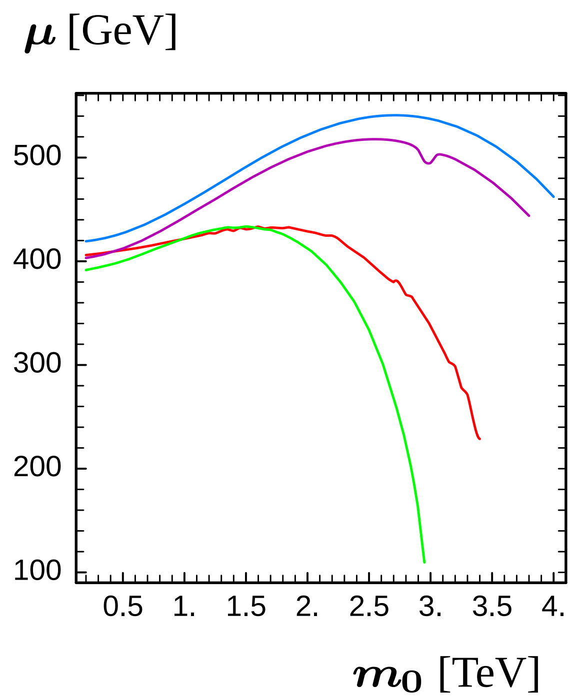
- 2-loop RGE's + 1-loop threshold corrections
- 1-loop corr. to top and bottom Yukawa couplings
- Higgs potential at 1-loop + leading 2-loop terms

Spectrum codes

Calculation of SUSY spectrum from high-energy input requires *sophisticated computational tools*.

Compare 4 public codes, Isasusy, Softsusy, Spheno, Suspect:

- typical differences in masses $\sim 1\%$
- can be larger in 'tricky' regions (large $\tan\beta$, large m_0)
- higher-order corrections ?
- bugs ?



[Allanach, SK, Porod, hep-ph/0302102]

`http://cern.ch/kraml/comparison`

Estimate of uncertainties

Theoretical / computational uncertainties have been estimated in two ways:

1. as the differences between state-of-the-art spectrum codes
   $\longrightarrow \boldsymbol{\Delta}_{codes}$ [AKP, hep-ph/0302102]
2. by varying the renormalization scale $\tilde{M}$ between 100 GeV and 1 TeV in one program (SPheno)
   $\longrightarrow \boldsymbol{\Delta}_{scale}$ [Allanach et al., hep-ph/0403133]

Neither approach is sufficient; can only give a rough estimate of the size of the uncertainties.

A more thorough investigation would be necessary....
see also Jack, Jones, Kord, hep-ph/0308231

Estimate of uncertainties, SPS1a

Sparticle	Isajet	Softsusy	Spheno	Suspect	Δ_{codes}	Δ_{scale}
$\tilde{\chi}_1^0$	95.47	97.33	97.18	97.53	1.03	0.34
$\tilde{\chi}_2^0$	181.68	181.09	180.85	181.19	0.42	1.1
$\tilde{\chi}_4^0$	376.07	384.08	383.01	384.75	4.34	0.3
h^0	113.78	111.01	111.06	112.87	1.38	1.2
A^0	394.93	401.19	400.59	402.19	3.63	0.7
$\tilde{e}_L$	204.65	204.26	207.14	204.07	1.53	0.31
$\tilde{e}_R$	143.11	145.53	143.94	145.40	1.21	0.82
$\tilde{u}_L$	564.66	567.14	565.29	571.95	3.65	9.1
$\tilde{u}_R$	548.26	546.88	547.62	551.91	2.51	8.4
$\tilde{g}$	611.74	608.78	604.07	611.21	3.83	1.2

Estimate of uncertainties, SPS1a

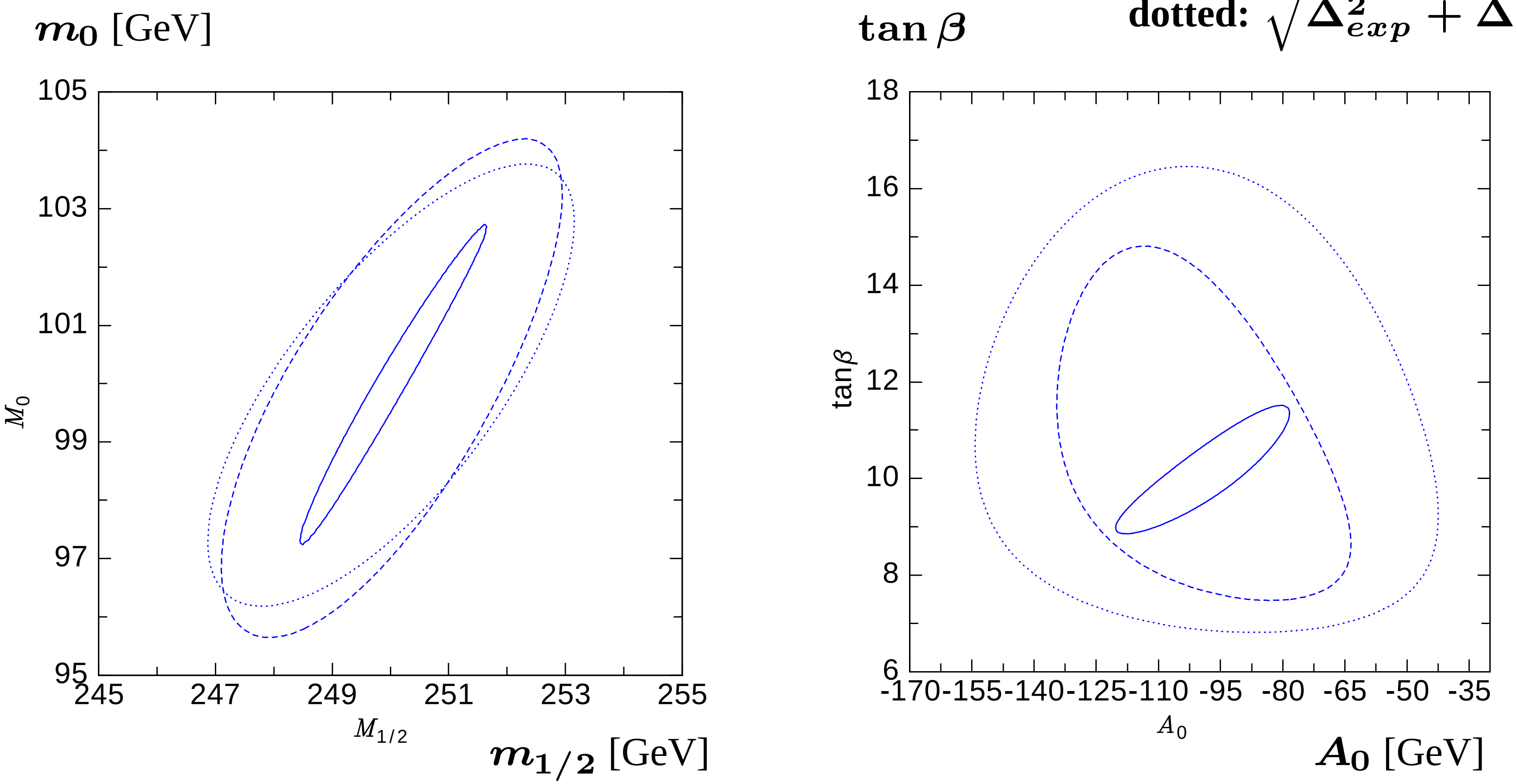
Quantity	Spheno	Δ_{exp}	Δ_{codes}	Δ_{scale}
m_{ll}^{max}	80.77	0.08	1.62	0.72
m_{llq}^{max}	454.10	4.5	2.76	8.1
m_{llq}^{min}	216.98	2.6	2.65	3.6
m_{lq}^{high}	397.21	3.9	3.64	7.7
m_{lq}^{low}	319.64	3.1	2.62	5.5
$m_{\tau\tau}^{max}$	83.56	5.1	1.30	0.8
m_{ll}^{max} $(\tilde{\chi}_4^0)$	284.51	2.3	3.72	0.7
m_{llb}^{min} $(\tilde{b}_1)$	195.97	4.1	2.77	2.9
$m_{\tilde{q}_R} - m_{\tilde{\chi}_1^0}$	450.30	10.9	2.42	8.1
$m_{\tilde{l}_L} - m_{\tilde{\chi}_1^0}$	109.96	1.6	1.71	0.23
$m_{\tilde{g}} - 0.99\, m_{\tilde{\chi}_1^0}$	507.86	1.3	4.68	1.3

Effect on top-down fit

solid: Δ_{exp}

dashed: $\sqrt{\Delta^2_{exp} + \Delta^2_{scale}}$

dotted: $\sqrt{\Delta^2_{exp} + \Delta^2_{codes}}$



$$m_0 = 100 \pm 2.8 \rightarrow 5, \quad A_0 = -100 \pm 34 \rightarrow 65,$$

$$m_{1/2} = 250 \pm 2.1 \rightarrow 4, \quad \tan\beta = 10.0 \pm 1.8 \rightarrow 6.$$

[fits by W. Porod]

Discrimination of models

Top-down fit ....

- has to be done on a model-by-model basis
- can favour or disfavour the tested model(s)
  e.g., fit of mGMSB to SPS1a data gives $\chi^2_{min}/14\,dof = 68$
- CANNOT exclude other possibilities

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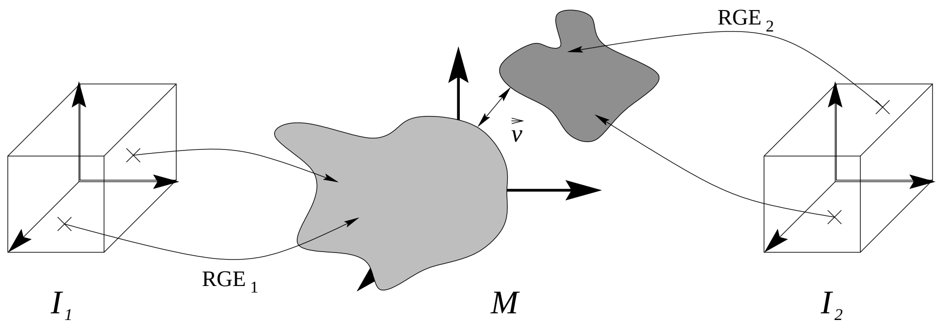
Interesting idea: **top-down mapping** of parameter space

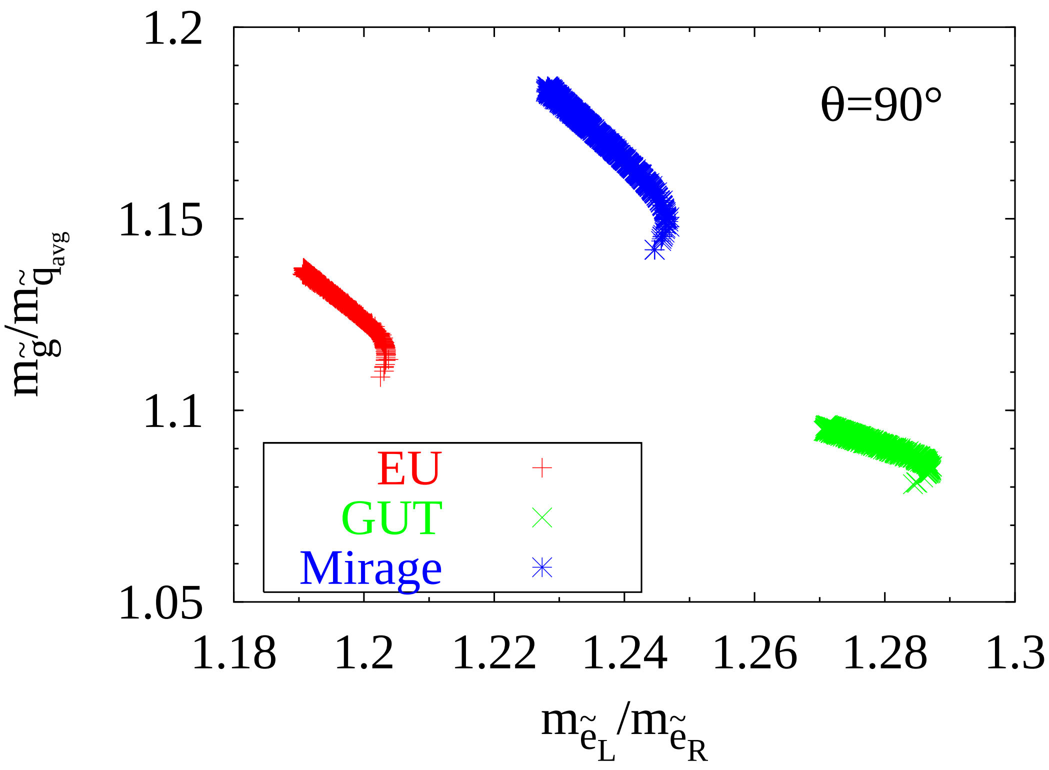


→ model ‘footprints’ of sensitive observables

[D. Grellscheid, hep-ph/0304277]

Discrimination of models

Example: discrimination of dilaton-dominated string scenarios by measuring $m_{\tilde{g}}/m_{\tilde{q}}$ and $m_{\tilde{e}_L}/m_{\tilde{e}_R}$ to $\sim 1\%$:



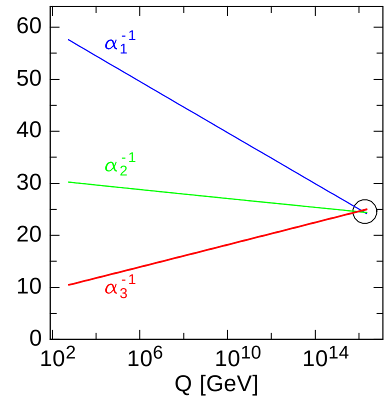
[D. Grellscheid, hep-ph/0304277]

LHC+LC:
bottom-up analysis

Bottom-up idea

Extract the SUSY Lagrange parameters from measurements at the EW/TeV scale and extrapolate them to the GUT scale.

[Blair, Porod, Zerwas, hep-ph/0007107]



- Model-independent test of high-scale boundary conditions!
- RGEs = highly coupled set of differential equations, at 2-loops "everything feeds into everything" ...

$\rightarrow$ needs almost complete spectrum information

$\rightarrow$ needs good precision to avoid that errors blow up

Precisions at LHC / LC

Examples of expected precisions at SPS1a:

	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	$\tilde{\chi}_1^\pm$	$\tilde{\chi}_2^\pm$	h^0	A^0
mass	97.2	180.7	364.7	381.9	179.7	382.3	110.8	399.4
LHC	4.8	4.7		5.1			0.25	
LC	0.05	1.2	3–5	3–5	0.55	3.0	0.05	1.5
LHC+LC	0.05	0.08	3–5	2.23	0.55	3.0	0.05	1.5

	$\tilde{e}_R$	$\tilde{e}_L$	$\tilde{q}_R$	$\tilde{q}_L$	$\tilde{t}_1$	$\tilde{b}_1$	$\tilde{g}$
mass	143.9	207.1	547.6	570.6	399.5	515.1	604.0
LHC	4.8	5.0	7–12	8.7		7.5	8.0
LC	0.05	0.2	–	–	2.0	–	–
LHC+LC	0.05	0.2	5–11	4.9	2.0	5.7	6.5

Can improve both LHC and LC mass measurements by coherent LHC/LC analyses! **[LHC/LC document]**

Lagrange parameters, EW scale

	ideal value	LHC+LC err.
M_1	101.66	0.08
M_2	191.76	0.25
M_3	584.9	3.9
μ	357.4	1.3
$M^2_{L_1}$	$3.8191 \cdot 10^4$	82.
$M^2_{E_1}$	$1.8441 \cdot 10^4$	15.
$M^2_{Q_1}$	$29.67 \cdot 10^4$	$0.32 \cdot 10^4$
$M^2_{U_1}$	$27.67 \cdot 10^4$	$0.86 \cdot 10^4$
$M^2_{D_1}$	$27.45 \cdot 10^4$	$0.80 \cdot 10^4$

	ideal value	LHC+LC err.
$M^2_{L_3}$	$3.7870 \cdot 10^4$	360.
$M^2_{E_3}$	$1.7788 \cdot 10^4$	95.
$M^2_{Q_3}$	$24.60 \cdot 10^4$	$0.16 \cdot 10^4$
$M^2_{U_3}$	$17.61 \cdot 10^4$	$0.12 \cdot 10^4$
$M^2_{D_3}$	$27.11 \cdot 10^4$	$0.66 \cdot 10^4$
$M^2_{H_1}$	$3.25 \cdot 10^4$	$0.12 \cdot 10^4$
$M^2_{H_2}$	$-12.78 \cdot 10^4$	$0.11 \cdot 10^4$
A_t	$-497.$	9.
$\tan\beta$	10.0	0.4

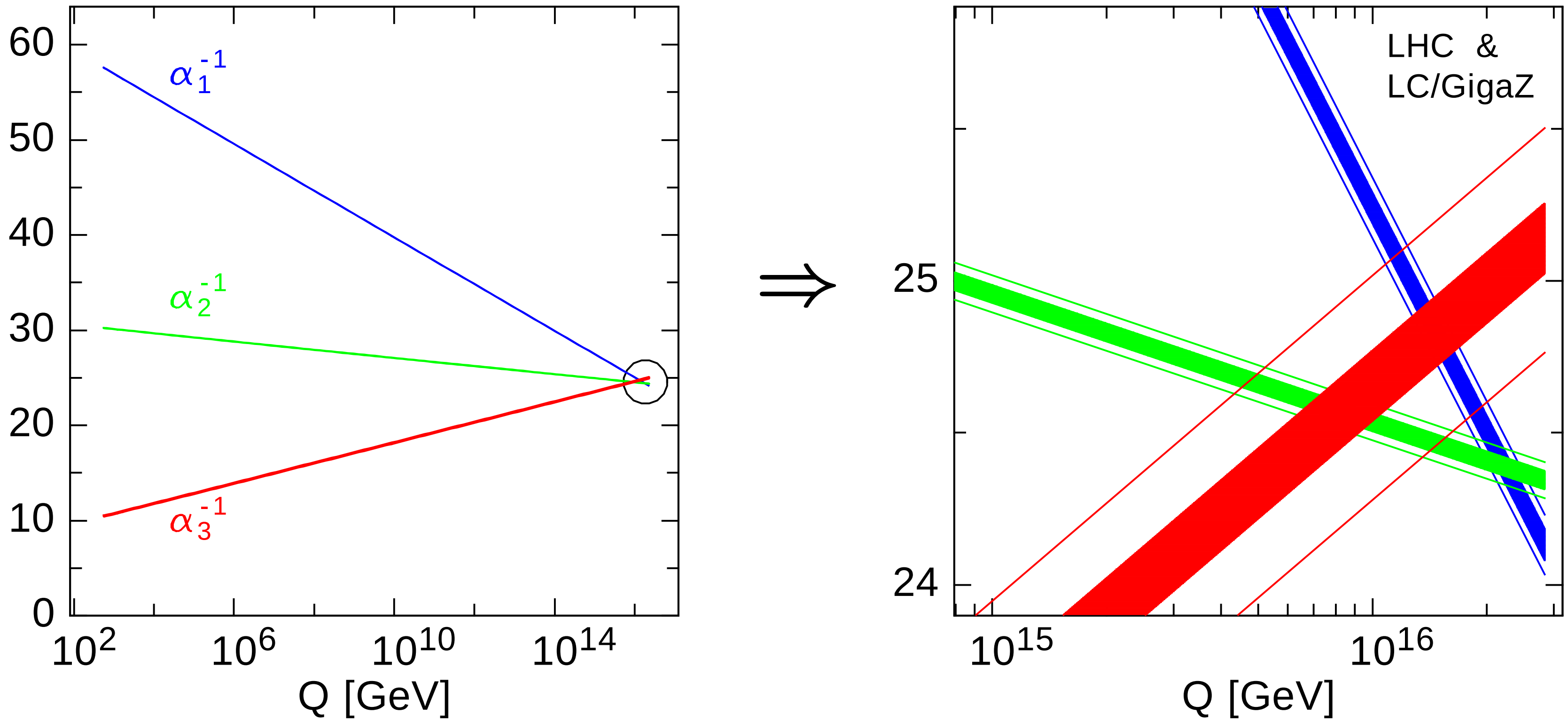
$\longrightarrow$ run parameters up to high-energy scale by RGEs

$\longrightarrow$ model-independent(!) test of boundary conditions

$\longrightarrow$ need BOTH LHC AND LC data for this exercise!

(complete spectrum contributes to 2-loop RG running)

Gauge coupling unification

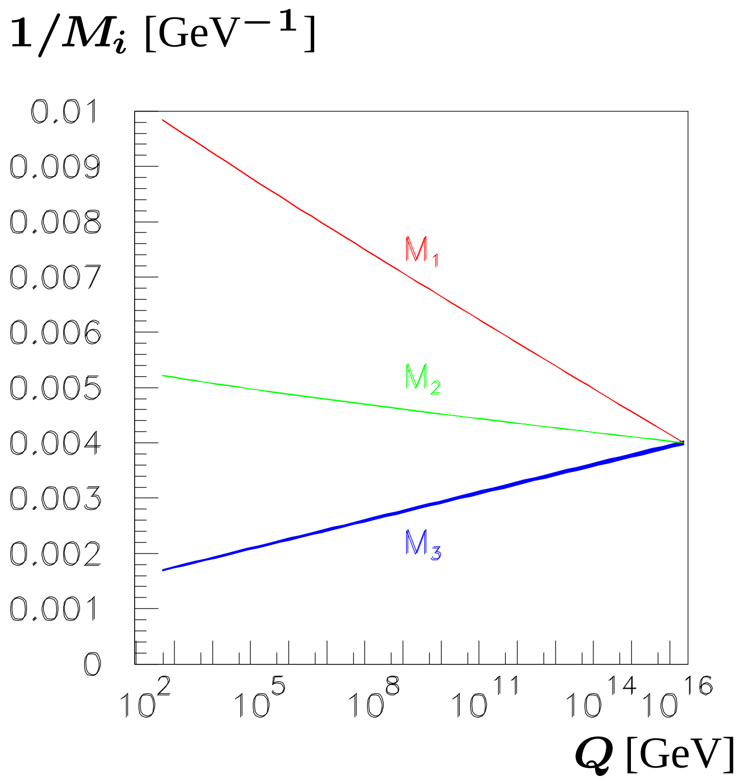


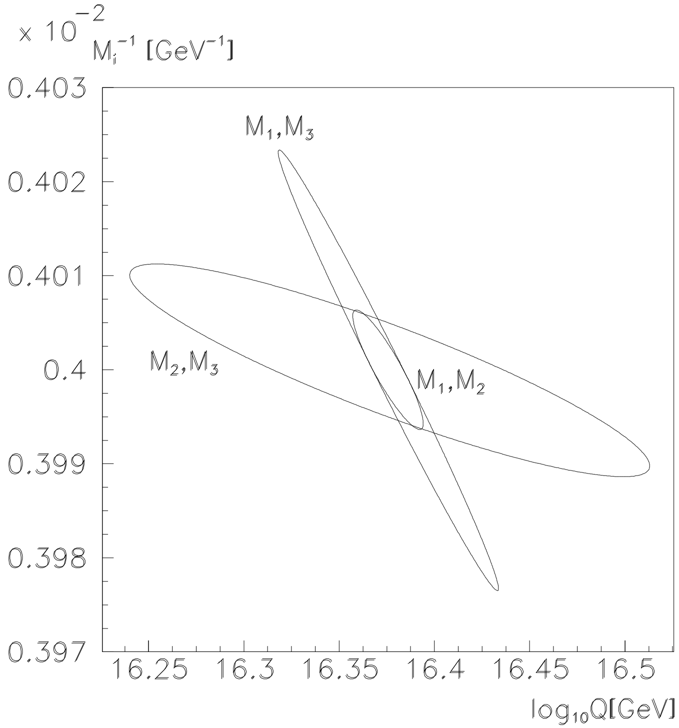
Running of the inverse gauge couplings; $\alpha_i = Z_i\,\alpha_U$.

Wide error bands: present data + LHC measurements.

Narrow bands: GigaZ + SUSY spectrum from LHC+LC.

Gaugino mass parameters



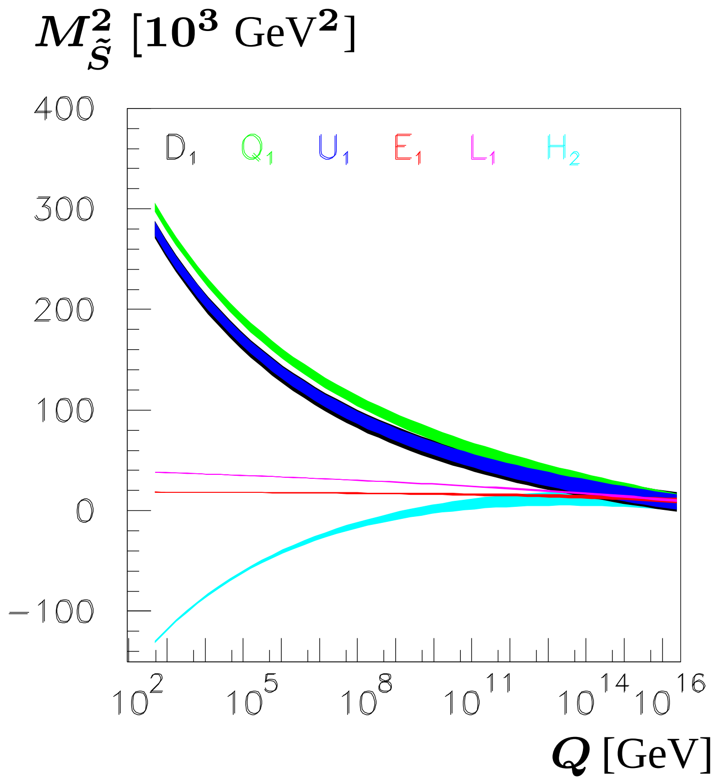


Evolution of the gaugino mass parameters M_i,

and the corresponding error ellipses of their GUT values.

$$\mathbf{M_1 = 250 \pm 0.15,\ M_2 = 250 \pm 0.25,\ M_3 = 250 \pm 2.3}\ \text{[GeV]}$$

Scalar mass parameters



$M_{\tilde{S}} = 100$ GeV

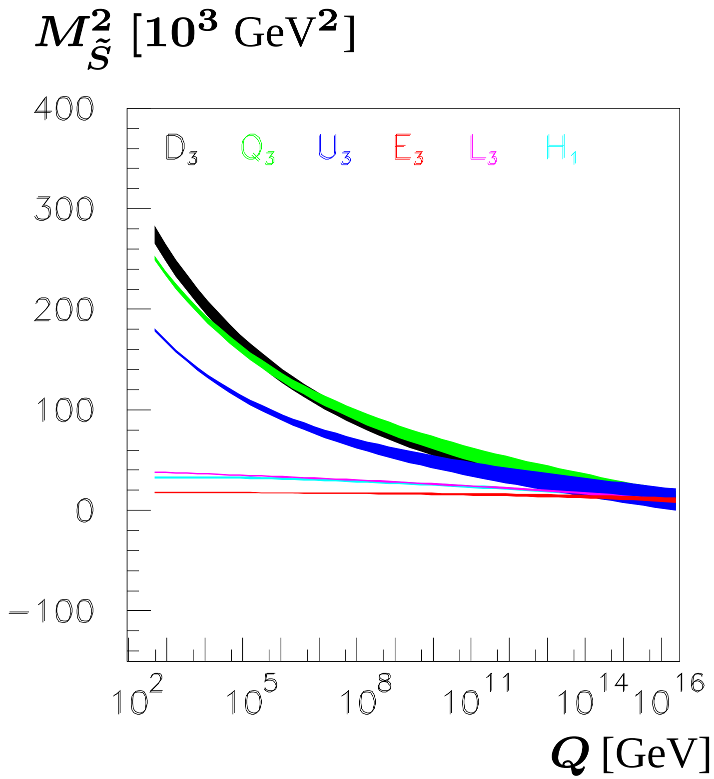
ΔM_{L_1}	$= 6$	ΔM_{L_3}	$= 7$
ΔM_{E_1}	$= 12$	ΔM_{E_3}	$= 14$
ΔM_{Q_1}	$= 23$	ΔM_{Q_3}	$= 37$
ΔM_{U_1}	$= 48$	ΔM_{U_3}	$= 58$
ΔM_{D_1}	$= 38$	ΔM_{D_3}	$= 42$
ΔM_{H_1}	$= 8$	ΔM_{H_2}	$= 41$

$A_t = -100 \pm 40$ GeV

model-independent test of scalar mass unification!

needs precise measurements of slepton, squark, gaugino and Higgs masses,
trilinear scalar couplings of 3rd gen. (A_t) $\Rightarrow$ **LHC+LC mandatory**

Scalar mass parameters



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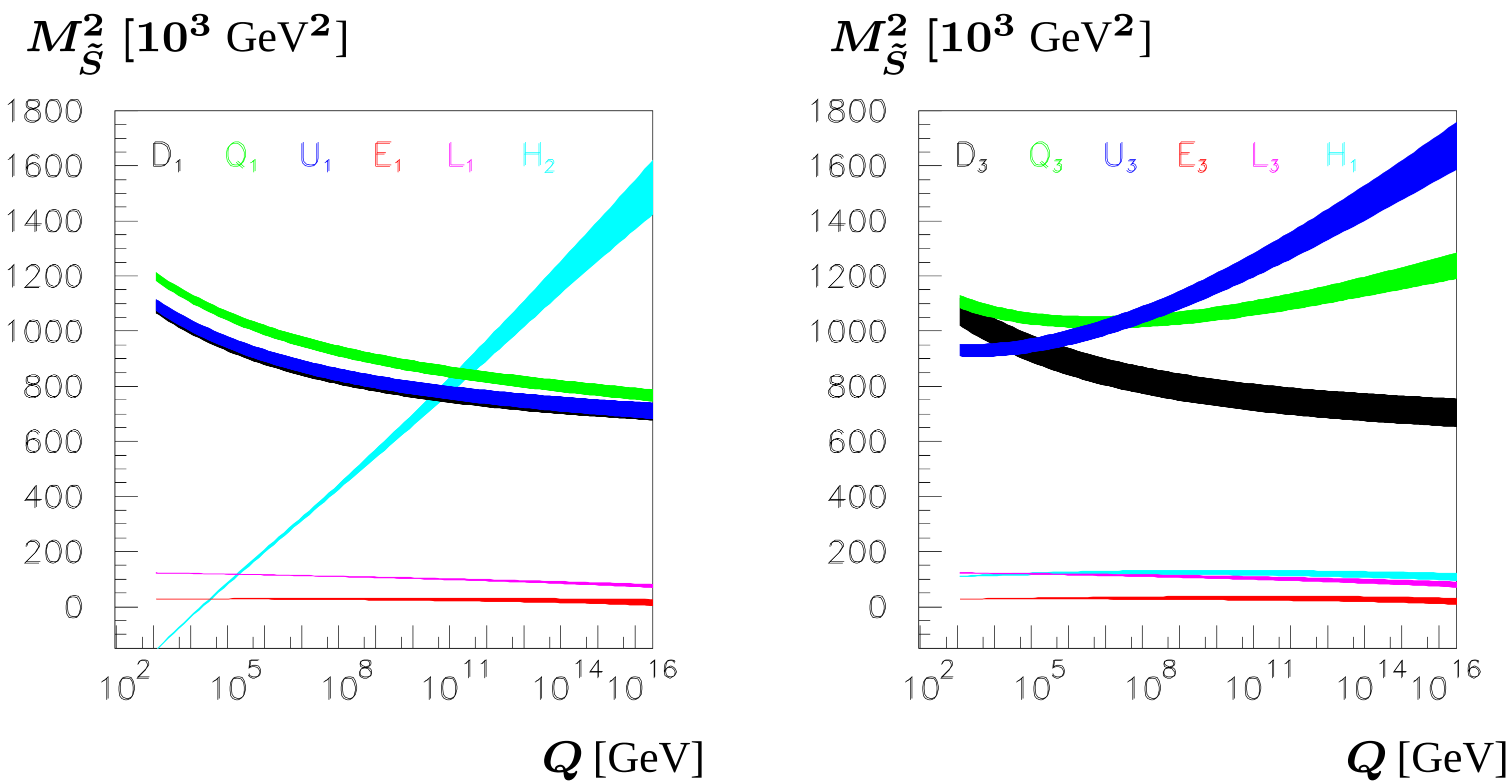
GMSB scenario



bottom-up evolution of a mGMSB scenario (SPS8)

clearly different pattern, can determine messenger scale

[Blair, Porod, Zerwas, hep-ph/0210058]

Conclusions

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Top-down methods are

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- very model dependent; need to identify sensitive observables for model discrimination

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The bottom-up approach

- allows for a model-independent reconstruction of the high-scale boundary conditions.
- requires a precise measurement of $\sim$ complete spectrum
  $\rightarrow$ coloured sparticles from LHC, weakly-interacting ones from LC
  $\rightarrow$ LHC/LC synergy

Conclusions -ff-

Uncertainties in SUSY spectrum computations are relevant

- at present, theoretical uncertainties are estimated to be of the order of LHC experimental acurracy
- need to improve existing calculations (higher orders ...)
- need a careful assessment of existing uncertainties for the reconstruction of the high-scale theory

Aim: match the experimental precisions at LHC/LC until data becomes available. Work to do for theorists ....